COMM 215: Business Statistics Solution to Practice Problems 2

Estimation and Hypothesis Testing

1 a) $H_0 : \mu = 105 \quad t_{.01,14 d.f.} = -2.264 \quad t = \dfrac{95.53 - 105}{15.39/\sqrt{15}} = -2.38$

$H_1 : \mu < 105$

$-2.38 > -2.626$ do not reject H_0

insufficient evidence to conclude that mean < 105

b) $t_{.05,14 d.f.} = -1.761 \quad -2.38 < -1.761$ you would reject H_0

c) $.01 < p < .025$

2 a) $n = \dfrac{(1.645)^2(.6)(.4)}{(.03)^2} = 721.6 = 722$

b) $n = \left(\dfrac{1.645(5)}{(.5)}\right)^2 = 270.6 = 271$

c) $110 \pm 1.96\left(6/\sqrt{70}\right)$, or 110 ± 1.406, or $(108.59, 111.406)$

3 a) $\begin{cases} H_0 : \mu = 10.8 \\ H_1 : \mu < 10.8 \end{cases} \quad t = \dfrac{10.2 - 10.8}{1.25/\sqrt{16}} = -1.92$; p-value: $0.025 < p < 0.05$

REJECT H_0 at $\alpha = .05$ level of significance.

b) $t_{.025,15 d.f.} = 2.131$ Since $-1.92 > -2.131$ DO NOT REJECT H_0

4 type I : Approving a below standard shipment

type II: Refusing a shipment that is up to standard

5 a) $73 \pm 3.182\left(\dfrac{1.414}{\sqrt{4}}\right) \quad 73 \pm 2.25$

b) $H_0 : \mu = 77$

$H_1 : \mu < 77$ reject H_0 if $t < -2.353$

$t = \dfrac{73 - 77}{1.414/\sqrt{4}} = -5.6577$

Reject H_0 and conclude that there is sufficient evidence to support the mean emission is less than 77 mg per cubic meter of exhaust.

6 a) H_0: $\mu = 5$ Reject H_0 if t >1.1812

H_1: μ>5

$$t = \frac{6.09-5}{6.41/\sqrt{11}} = 0.564.$$ DO NOT Reject H_0.

Insufficient evidence to support the airline's claim. p-value >.10

b) $6.09 \pm 2.228\left(\frac{6.41}{\sqrt{11}}\right)$ 6.09 ± 4.306 (1.784, 10.396)

7 $H_0: p = 0.04$ Reject H_0 if z<-2.33

H_1 :p<0.04 $$z=\frac{.10-.04}{\sqrt{\frac{(.04)(.96)}{80}}}=2.74$$

Since 2.74 > -2.33, DO NOT reject H_0

No evidence that p< 0.04. p-value= p(z<2.74)=.9969

8 $H_0: p = .51$

H_1 :$p \neq .51$

$p=\frac{690}{1335} = .5169$ Reject H_0 if $z>|1.645|$

$$z = \frac{.5169-.51}{\sqrt{\frac{(.51)(.49)}{1335}}} = .5044.$$ DO NOT REJECT H_0. Insufficient evidence. Reject researcher's claim.

9 Assume standardized test scores follow a normal distribution with μ=100, σ=10.

$\begin{cases} H_0: \mu=100 \\ H_1: \mu>100 \end{cases}$ Test Statistic: $z=\frac{\bar{X}-100}{10/\sqrt{n}}$; Rejection region: $Z > Z_{0.025}=1.96$

with n=25, $\bar{X}$ = 103, $z=\frac{103-100}{10/\sqrt{25}} = +1.5$. Since $Z=1.5 < Z_{0.025}=1.96$, do not reject at α=2.5%.

Insufficient evidence to support the claim of above average intelligence. p-value = 0.0668.

10 a) $n=\frac{(2.58)^2(.5)(.5)}{(.02)^2} \approx 4160.25$; n=4161

b) $0.48 \pm 2.58\sqrt{\frac{(.48)(.52)}{4161}}$, or 0.48 ± 0.01998, or (.46, .50)

11 a) $60 \pm 1.96\left(7.5/\sqrt{100}\right)$

$60 \pm 1.47 \rightarrow 58.53\%$; 61.47% true mean falls between 58.53% and 61.47% 95% of the time

b) $.07 \pm 1.96\sqrt{\frac{(.07)(.93)}{100}} \rightarrow .07 \pm .05 \rightarrow (0.02, 0.12) \rightarrow$ 95% confidence

True proportion of failing is between 2% and 12%

c) $n = \frac{(2.575)^2(7.5)^2}{(5)^2} = 14.92 = 15$ and $n = \frac{(2.575)^2(.07)(.93)}{(.1)^2} = 43.7 = 44$

To satify both conditions n must be at least 44.

12 a) *i*) $H_0: p = 0.25$ Reject H_0 if $z > |1.96|$

H_1 :p $\neq 0.25$

$$\hat{p} = \frac{15}{35} = 0.4286 \qquad z = \frac{.4286-.25}{\sqrt{\frac{(.25)(.75)}{35}}} = 2.44$$

Since 2.44>1.96 reject H_0 and conclude that proportion using visual basic is different from 25%.

ii) p-value=P($z \geq 2.44$)$\times$2=.0073$\times$2=0.0146

b) *i*) average cost > \$40,000 if average day > $\frac{40000\text{-}10000}{1200} = 25$

$H_0: \mu = 25$ Reject H_0 if $Z > Z_{0.05}$=1.645

$H_1: \mu > 25$

$$z = \frac{27.2-25}{5.5/\sqrt{35}} = 2.37$$

Since Z=2.37 > $Z_{0.05}$=1.645 Reject H_0. Conclude average day >25.

The evidence is sufficient suggesting that mean will exceed \$40,000.

ii) p-value P($z \geq 2.37$)=.5-.4911=0.0089.

13 a) $\bar{x}$=237.829 s=36.369

$$237.83 \pm 1.645\left(\frac{36.37}{\sqrt{35}}\right) \qquad 237.83 \pm 10.11$$

(227.72, 247.94)

b) $\hat{p}$=more than 240 seconds $= 15/35 = 0.4285$

$$0.43 \pm 2.575\sqrt{\frac{(.43)(.57)}{35}} \quad , \quad \text{or} \quad 0.43 \pm 0.2155, \text{ or } (0.2145, 0.6455)$$

14 $$n = \left(\frac{1.96(175.5)}{20}\right)^2 = 295.8 \rightarrow 296$$

15 $$n = \frac{(1.88)^2(.5)(.5)}{(.055)^2} = 292.099 \rightarrow 293$$

16 i) $\hat{p}=\frac{800\text{-}240}{800}=0.70$ $\sigma_{\hat{p}}=\sqrt{\frac{(.70)(.30)}{800}}=0.0162$

$0.70\pm2.33(.0162)$ or 0.70 ± 0.0377 (.6623, .7377)

ii) $2.45\pm1.96\left(1.3/\sqrt{800}\right)$ $2.45\pm.09$ (2.36, 2.54)

17 $n=\left(\frac{2.575(300)}{45}\right)=294.65=295$

18 a) $18.30\pm t(.025,\ 8\ d.f)\left(6.3/\sqrt{9}\right)$

18.30 ± 4.8426 (\$13.46, \$23.14)

b) normally distrubuted population

c) $\hat{p}=\frac{49}{80}=0.6125$

$0.6125\pm2.17\sqrt{\frac{.6125(.3875)}{80}}$

$0.6125\pm.1182$

(0.4943, 0.7307)

d) $n=\left(\frac{(1.96)(6.30)}{1.00}\right)^2=152.47$, the sample size should be increased to 153.

19 a) $H_0:\mu=2.8$ Reject H_0 if $z<-1.645$

$H_1:\mu<2.8$ or if $t<-1.66$

$z=\frac{2.61-2.8}{0.9/\sqrt{100}}=-2.11$

Since $-2.11<-1.645$ Reject H_0 significant evindence that

$\mu<2.8$ and therefore promotion was not profitable.

b) p-value $p(z<-2.11)=0.5-0.4826=0.0174$

or if t $.01<p<.025$

c) Type I since you are rejecting

d) $s/\sqrt{n}=.9/\sqrt{100}=.09$

reduced by half $=.09/2=.045$

$.9/\sqrt{n}=.045$ so $n=400$